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THE IMPACT OF MONETIZATION ON THE TUNISIAN ECONOMY USING A SIMPLE DSGE MODEL

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Abstract *The presence of money in the economy is mostly considered obvious. This paper investigates the effect of monetization on the Tunisian economy with a Dynamic Stochastic General Equilibrium (DSGE) model. The article also intends to show the main driving forces behind Tunisian business cycle and to understand how the productivity and monetary shocks impact and transmit into the Tunisian economy. Two Small Open Economy Neo-Keynesian (NK) models with price stickiness without and with money are carried out in this study. The results show that the supply shock has a positive effect on the macroeconomic variables. Nevertheless, the monetary shock has negatively affected the variables, especially the output, the capital and wages.*

Keywords:- Monetization, money in utility, Tunisia, DSGE models, monetary policy.
JEL Codes: E4, E5

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Introduction:

The presence of money in the economy is mostly considered obvious. Central Banks in developed and developing countries consider that cash gives people the freedom to opt the way that is convenient for them to pay.

In Tunisia, the Central Bank and the banking sector, together, have a central duty to ensure the smooth supply of cash and facilitate the use of cash in payments by people and businesses. Moreover, they ensure that it remains widely available and accepted.
Dinar banknotes and coins are legal tender in the country and cash is a public form of payment to which every person (citizen and resident) have access.

According to the latest banking supervision report, recently published by the Central Bank of Tunisia (CBT), in 2020, the most of transactions were done by cash and only 17% of Tunisians used at least once a month a means of payment replacing cash, mainly bank cards. Despite the development observed, the activity of electronic payments in Tunisia has remained weak.

This paper investigates the impact of monetization on the Tunisian economy through a Dynamic Stochastic General Equilibrium (DSGE) model with both the integration of money in the utility function and of monetary authority. The paper also intends to show and to understand how the shocks and monetary policy transmit into the Tunisian economy. This study contributes to the literature in at least two ways about the relationship between the money demand function and macroeconomic variables. It concentrates also on the impact on the Tunisian economy about which few studies have been undertaken recently.

The next section presents a literature review. The second section defines a brief history and the Tunisian policies of monetization. The first simple DSGE model without money is presented in the third section. The last one provides the second empirical example of monetization on the economy of Tunisia in a simple model with money in utility function and with a monetary authority.

Literature review

A large body of literature exists in estimating money demand functions. Kremer et al. (2003) criticized previous work that have come to the conclusion that real money balances don't play any role in estimated NK models. They studied the role of real balances in DSGE model applied to German data with a simple NK model with non-separable preferences in real balances and consumption. Results showed that real money balances play an active role in the determination of inflation and the dynamics of the output.

Unlike Ireland (2004), a recent contribution of Şahin & Koç (2019) found that money has a role in the monetary business cycle model. Estimation results show that in the short run, the monetary authority is not so reactive to inflation and output gap. Also, Castelnuovo (2012) uses a sticky price NK model with money. It is deduced that money has a significant role in US business cycles and that this role varies over time.

Several Tunisian works which are based on the study of money demand was carried out. Daboussi & Majoul (2014) criticized the effectiveness of monetary targeting in Tunisia and presented the limits of the adoption of this policy. At the same time, they presented solutions in order to pass to a stricter policy.

Abdelli & Belhadj (2015) aimed to highlight the monetary transmission mechanisms and to understand how shocks and monetary policy of inflation targeting transmit into the Tunisian economy with NK model with real balances in the utility function. Their main findings showed that the currency has effects on the output and interest rate in the short run only.

As far as MIU theory is concerned, Goyal & Kumar (2018) build a MIU model to identify and check for money on output separately from the interest rate. In an economy composed of representative household, wholesale and retail firms, and a central bank. Impulse responses showed that real balances do affect output and inflation in India which supports the evidence from a NK model

Monetization phenomenon in the Tunisian economy

The actual definitions of money used by governments and central banks vary from country to another. Essentially, every central bank in the world uses some forms of measure for narrow money, broad money and other intermediate items.

In Tunisia, the CBT provides four measures of money: M1, M2, M3, and M4, where M1 is the narrowest and M4 is the broadest.

Monetization is the introduction of new forms of payment into the economic circuit by commercial banks or central banks by the simultaneous increase of their assets and their liabilities (new currency issued in return of the debt).

Chandavarkar (1977)3defines monetization as “the enlargement of the sphere of the monetary economy”. In other words, the rate to which money is used as a medium of exchange, a unit of account and a store of value. The author claims that, monetization is one of the most significant manner of the development and growth of the less developed countries’ economies.

The Federal Reserve defines money supply as the total amount of money—cash, coins, and balances in bank accounts—in circulation. By way of explanation, the money supply is usually defined as a set of safe assets that households and businesses can use to pay or hold as short-term investments.

Monetization can take different forms referring to the context; (1) economic policy context and (2) digital economy context.

The first one points out monetization by commercial banks by increasing the monetary creation available in the economy and monetization by central banks by the increase of liquidity available for commercial banks through the raise of the monetary basis (Central Bank currency)4.Debt monetization or monetary financing is a second form of monetization by central banks that is the practice when a government borrows money from the Central Bank in order to finance public spending on behalf of selling bonds to the private sector or increasing taxes.

The second one permit households to get rewarded for the ways they build their communities via social media. In other words, monetization converts non-revenue generating items to cash flows. When people navigate websites and click on advertiser links placed in the platform’s content, website owners win money. For example, in music sites, advertiser links are put either before or after the music. Website owners can be paid for the number of times the visitors to the site see ads without participating with them, depending on the arrangements with advertisers. The money paid by advertisers, if a website attracts enough visitors, can increase considerable revenue.

[3] McLoughlin, C., and Kinoshita, N. (2012). *Monetization in Low- and Middle-Income Countries* (IMF Working Paper No. 12/160).

[4] Refers to notes and coins in circulation in addition to commercial bank’s reserves. Generally, the aggregate M1 is called monetary base.

Website and mobile apps that generate revenue are monetized via online advertisements.

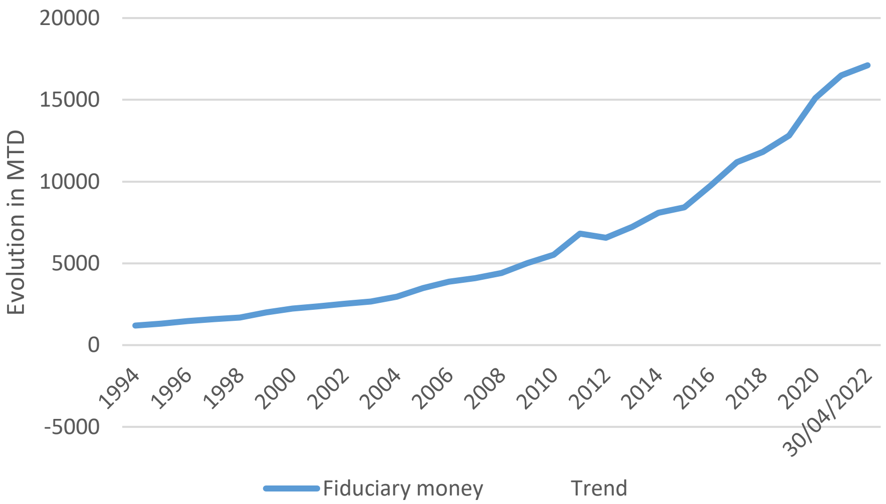
In order to explore the monetization in Tunisia, the next section deals with a short history of the introduction of coins and banknotes. It treats also the main policies adopted by the CBT to deal with this phenomenon.

Monetization phenomenon in Tunisia

Brief history

The Tunisian Dinar (TND) is the Tunisian currency since November, 1st, 1958, by virtue of law n°58-109 of October 18, 1958, on monetary reform, although it did not start to be used until 1960. Banknotes and coins in circulation have followed an upward trend from 1994 until now, a surge from 1196 MTD to 17111 MTD (figure 1-1).

Figure 1-1: Evolution of total fiduciary circulation (1994-2022)



Source: Graph elaborated by the author using data from CBT

Total fiduciary circulation reached 15,749.4 MD at the end of 2020. An increase of 16.6%, compared to 2019, with a predominant share of banknotes (97.4%), according to the CBT, for fiscal year 2020.It should be noted that the rate of increase in currency circulation has accelerated, since it only increased between 2018 and 2019 by 8.5%.The 20 dinar banknotes in circulation occupy the first place at more than 9,410 MD, followed by the 10 dinar banknotes (3,825 MD), the 50 banknotes (2,059 MD) and the 5 dinar banknotes (45 MD).

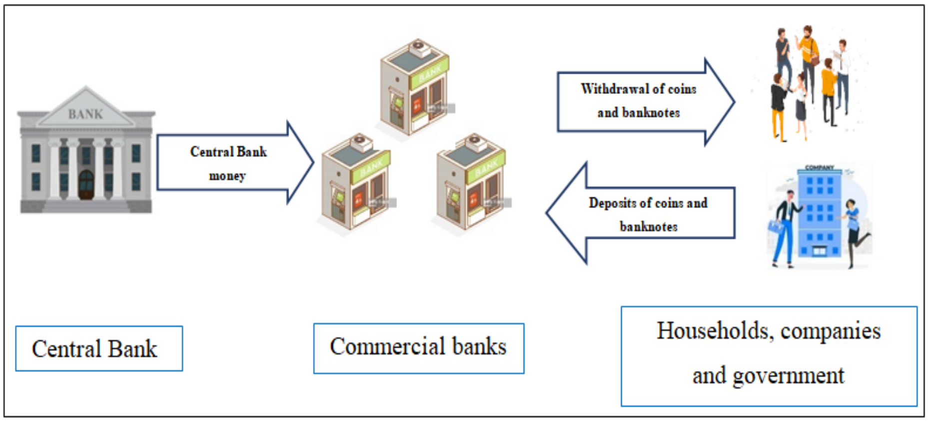
For coins, 1000 millime coins in circulation are estimated at more than 143 MD, followed by 5-dinar coins (over 100 MD), 500 millime (over 56 MD) and 2 dinars (over 45 MD), reflecting the importance of use of cash as the preferred method of payment. Indeed, during the last decade, it has recorded an average growth rate of 8%.

Owing to recent CBT data, coins and banknotes in circulation gained 17111 MTD in April, 2022, as the highest level until today.

Actors of monetization in Tunisia

Monetization is managed by the CBT, 23 commercial banks and 1043 post offices. The central bank exercises on behalf of the State an exclusive privilege of issuing money on the territory of the republic (figure 1-2).

Figure 1-2: Actors of monetization



Source: Author's elaboration.

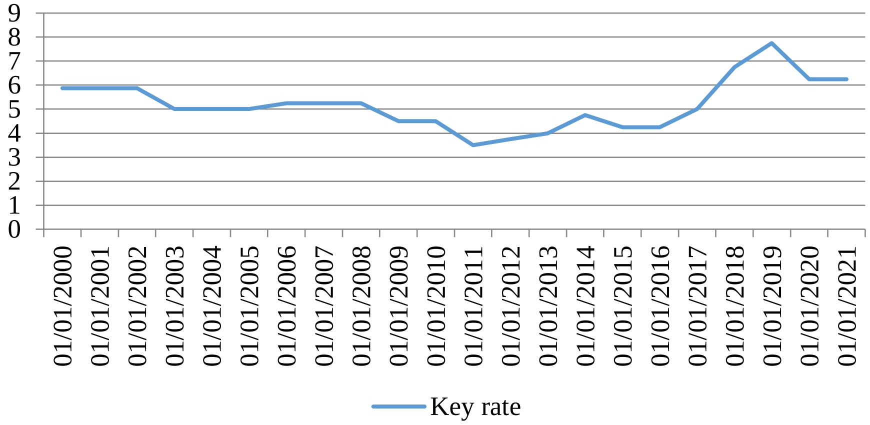
The CBT have the sole right to issue fiduciary money. Banknotes and coins follow a specific path through the economy managed by a precise policy.

Policies of monetization in Tunisia

In Tunisia, the CBT prints new banknotes and mint coins for a simple renewal of the stock of used and destroyed banknotes and deteriorated coins or due to worn category of banknotes with the aim of improving the quality of banknotes and coins in circulation. The CBT issues new coins and banknotes also in the presence of a massive counterfeiting or even after a major political event (such as the revolution in January 2010).

The choice of suppliers to print banknotes or mint coins is made through extensive international consultations among those who meet the technical, financial and support requirements recommended. The costs of raw materials influence the choice of alloys for making coins, cotton and ink for banknotes. From 1990 the CBT carried a discretionary monetary policy until September, 1958 after the CBT was created. From 2000 to 2020, the CBT drove an expansionist monetary policy at different times for multiple reasons. In 2003, the CBT dropped its key interest rate on two separate occasions from 5.875% to 5.5% then to 5% on June (CBT, 2003). In order to promote Investment and to create new job opportunities, the CBT dropped its key rate to 4% in 2009. This coincided with an increase by 13.7% of fiduciary money in circulation from 2008 (CBT, 2009).

Figure 1-3: Expansionist monetary policy driven by CBT (2000-2021)



Source: Graph elaborated by the author using data from CBT.

As the figure 1-3 shows, the key interest rate was at its lower value in 2011 (3.5%). After the revolution and with the economic recession of 2011, the CBT lowered its key rate twice because of the near-stop of production and export in the key sectors of the country's economy (industry, energy and tourism) and for fear of seeing the productive structures of the economy collapse (Alimi, 2019). The monetary authority reduced its rates from 4.5% to 4% in June and by half a percentage point to bring it down to 4% in September, 2011. Moreover the CBT injected 3,616 MTD to provide liquidity to the banks. In this context, banknotes and coins in circulation registered a record growth of 22.5% in 2011 (CBT, 2011).

In support of the pandemic containment measures, a package of measures has been implemented in order to limit its impact both on needy families and on the productive sectors. The key interest rate was dropped to 6.25% from 7.75% in 2020 (CBT, 2020).

The Board of Directors of the CBT decided on May 17, 2022, to raise the key rate of the BCT by 75 basis points, bringing it to 7% after having been stable the first 5 months of the year. This raise aimed to tackle inflationary pressures and especially to slow down its acceleration that represents a risk to the economy and a threat to purchasing power.

The construction of a simple DSGE model without money: The case of Tunisia

Dynamic Stochastic General Equilibrium (DSGE) modeling is a macroeconomic method used by monetary and fiscal authorities for policy analysis for historical and forecasting purposes. These dynamic models take into account time, forecast unexpected things that might happen and take into account the system as a whole (households, firms, government). Those models have become very popular in recent years and a point of reference in modern macroeconomics because of their positive contributions to discussions about problems and their usefulness in forecasting.

In a framework where flexible-price real money balances enter the production function, Benchimol (2015) departed from a NK DSGE model inspired by Gali (2008) to check whether an increase in the real money supply increases the productive capacity of the economy. The simulations showed that the firm money shock has an important influence on flexible-price output and a significant impact on output, and likewise, the estimated contribution of the firm's money holdings appears to be significant, implying that this shock might potentially affect monetary policy.

Our basic New Keynesian model without money

This section seeks to present different equations of the model (households, retailers and wholesalers) inspired from Junior (2016) and Romer (2011) that will be modeled to Tunisia introducing price stickiness and monopolistic competition.

A simple NK model with Small Open Economy (SOE) will be considered. It will be also considered that prices are sticky i.e. prices remain constant or adjust slowly.

Households

Our SOE is formed by a unitary set of households indexed by j ϵ [0,1] whose problem is to maximize the following welfare function additively separable into Consumption (C) and Labor (L) :

$$\max_{C_{j,t}} \quad E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right) \quad (1.1)$$

Where E_t is the expectations operator, β is the intertemporal discount factor, C is goods consumption, L is the number of hours worked, σ is the relative risk aversion coefficient and φ is the marginal disutility in respect of labor supply.

The utility function is characterized by $U_C > 0$ and $U_L < 0$, this implies that, respectively, consumption and labor have positive and negative effects on household's utility. Further, $U_{CC} < 0$ and $U_{LL} < 0$ so that the utility function is concave.

Households maximize their welfare function subject to intertemporal budget constraint, indicating the resources available and how they are allocated. The intertemporal budget constraint of the households can be written as follows:

$$P_t\,(C_{j,t} + I_{j,t}) = W_t\, L_{j,t} + R_t\, K_{j,t} + \Pi_t \quad (1.2)$$

Where P is the general price level, I is the level of investment, W is the level of wages, K is the capital stock, R is the return on capital and Π is the firms' profits.

Another important component is the capital accumulation equation which shows how the capital stock changes over years:

$$K_{j,t+1} = (1 - \delta)\, K_{j,t} + I_{j,t} \quad (1.3)$$

δ is the physical depreciation rate, so that between t and t+1, δk_t units of capital are lost from depreciation. However, in year t it is investment that allows new capital in the following year.

Equations (1.1), (1.2) and (1.3) establish the households' problem which the following Lagrangian will be used in order to solve it:

$$\mathcal{L} = \mathrm{E_t} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right) - \lambda_{j,t} \left(P_t C_{j,t} + P_t K_{j,t+1} - P_t(1-\lambda)K_{j,t} - W_t L_{j,t} - R_t K_{j,t} - \Pi_t \right) \right)$$

(1.4)

λ refers to the Lagrange multiplier.

When deriving the Lagrange to consumption, labor and capital accumulation, first-order conditions are obtained:

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0$$

(1.5)

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -L_{j,t}^{\varphi} + \lambda_{j,t} W_t = 0$$

(1.6)

$$\frac{\partial \mathcal{L}}{\partial K_{j,t+1}} = -\lambda_{j,t} P_t + \beta \, \mathrm{E_t} \, \lambda_{j,t+1} [(1-\sigma) \, \mathrm{E_t} \, P_{t+1} + \mathrm{E_t} \, R_{t+1}] = 0 \qquad (1.7)$$

Solving for λ_t equations (1.5) and (1.6), the household's labor supply equation is obtained:

$$-C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$$

(1.8)

Where $-C_{j,t}^{\sigma} L_{j,t}^{\varphi}$ is the Marginal Rate of Substitution (MRS) of consumption-Leisure and $-\frac{W_t}{P_t}$ is consumption-leisure relative price which refers also to the real wage.

This equivalence shows the existence of a tradeoff between labor and leisure. More hours worked earn higher incomes only if workers diminish hours of leisure they enjoy. Even though, the more real wages increase the more consumption increase without the need to renounce leisure.

Euler equation is the last equation for the household; it allows highlighting today's consumption compared to of that of the future, in other words determines his savings' decision(it is the acquisition of goods in this models), so the household wonders if he will consume today or he will consume in the future (saving at t). Euler equation is found after determining from equation (1.5) $\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}$ and $\lambda_{j,t+1} = \frac{C_{j,t+1}^{-\sigma}}{P_{j,t+1}}$, then using these results in equation (1.7):

$$\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$

(1.9)

Thus, a surge in interest rate expectations will make the present consumption more expensive and, ceteris paribus, future consumption will rise.

It will be easier to assume that $\beta = 1$ and $\delta = 1$,

$$-E_t\left[\frac{1}{\Pi_{t+1}}\left(\frac{C_{j,t+1}}{C_{j,t}}\right)^{\sigma}\right] = -E_t\left(\frac{r_{t+1}}{\Pi_{t+1}}\right)$$

With:

$-E_t\left[\frac{1}{\Pi_{t+1}}\left(\frac{C_{j,t+1}}{C_{j,t}}\right)^{\sigma}\right]$: Present-future MRS,

$-E_t\left(\frac{r_{t+1}}{\Pi_{t+1}}\right)$: Relative price of present-future consumption,

$E_t\, r_{t+1} = E_t\left(\frac{r_{t+1}}{P_{t+1}}\right)$: real rate of return on capital.

To sum things up, the problem of the household comes down to options. The first is an intertemporal choice between possessing consumption and leisure goods. The second is an intratemporal choice between present and future consumption.

1.1.1 Retail firms

The production sector of the economy is divided into two parts; ones that produce intermediate goods called wholesale firms and others that produce final goods known as retail firms. It is assumed that the market is in monopolistic competition.

The retail firms sector is characterized by a single firm that adopts a pertinent technology to turn on intermediate goods to a single consumable good.

The aggregate good is produced by passing by different steps; the retailer must buy inputs (a quantity of goods) from the wholesaler companies then transform them into a bundle of goods that will be sold to the final agent. The idea is that retailers regroup a large number of goods to a unique good to be consumed.

Celso (2016), with the aim of representing the problem of retailers, used the Dixit and Stiglitz aggregator (1977) to describe the aggregate technology:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}}\, dj\right]^{\frac{\psi}{\psi-1}} \qquad (1.10)$$

Where Y_t is the product of retailers at t, $Y_{j,t}$ is the wholesale good j for j ϵ [0, 1], and ψ is the elasticity between wholesale goods, $\psi > 1$.

The problem of the retail firm is to maximize its profit function which is presented as follows:

$$\max_{Y_{j,t}} \qquad P_t Y_t - \int_0^1 P_{j,t} Y_{j,t}\, dj \qquad (1.11)$$

With P_t the nominal price of retail product and $P_{j,t}$ the nominal price of wholesale good j.

Substituting (1.10) in (1.11):

$$\max_{Y_{j,t}} \qquad P_t \left[\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}}\, dj\right]^{\frac{\psi}{\psi-1}} - P_{j,t} \int_0^1 Y_{j,t}\, dj \qquad (1.12)$$

With a little algebraic transformation, the first-order condition is:

$$\left[\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}}\, dj\right]^{\frac{1}{\psi-1}} Y_{j,t}^{\frac{-1}{\psi}} - P_{j,t} = 0$$

The aggregate technology (equation (1.10)) may be written as follows:

$$Y_t^{\frac{1}{\psi}} = \left[\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}}\, dj\right]^{\frac{1}{\psi-1}}$$

This form is more helpful with the objective to find the demand function for wholesale good j. After eliminating the right-hand side of the above equation and raising the result to the power of $-\psi$, the expression of the demand function for wholesale good j is achieved:

$$Y_{j,t} = Y_t \frac{P_t}{P_{j,t}}^{\psi}$$

(1.13)

It is directly proportional to aggregate demand Y_t and inversely proportional to its relative price level $\frac{1}{\frac{P_{j,t}}{P_t}}$.

The mark-up rule for final goods is got by substituting equation (1.13) in equation (1.10):

$$P_t = \left[\int_0^1 P_{j,t}^{1-\psi}\, dj\right]^{\frac{1}{1-\psi}}$$

(1.14)

Wholesale firms

The retail firms sector is characterized by a multiple number of firms producing every one differentiable goods. By a production function, these firms must decide the quantity of factors of production to be used in order to the goods' prices.

Firms that produce intermediate goods have some degree of market power, they are price setters and they have constant returns to scale this means that whatever the quantity produced could be it will drive to a marginal production cost.

The intermediate producers solve their problem in two phases; in the first one, the firm serves of the prices of the factors of the production and determines the amount of capital and labor that minimizes its total production cost. In the second phase it fixes the prices at which the goods will be sold.

Firstly, the firm minimizes its total production cost:

$$\min_{L_{j,t}K_{j,t}} \qquad W_t L_{j,t} + R_t K_{j,t}$$

(1.15)

Subject to the following technology:

$$Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$$

(1.16)

With the low of motion f productivity:

$$\log A_t = (1-\rho_A)\log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t \qquad (1.17)$$

With A_{ss} is the value of productivity at the steady state, ρ_A is the autoregressive parameter of productivity ($|\rho_A| < 1$) and $\epsilon_t \sim N(0, \sigma_A)$.

The Lagrangian is presented as follows aiming to solve the wholesale firm's problem:

$$\mathcal{L} = W_t L_{j,t} + R_t K_{j,t} + \mu_{j,t} \left(Y_{j,t} - A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}\right) \quad (1.18)$$

Solving the previous problem we arrive to the first order conditions:

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = W_t - (1-\alpha)\mu_{j,t} A_t K_{j,t}^{\alpha} L_{j,t}^{\alpha} = 0 \quad (1.19)$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t}} = R_t - \alpha\, \mu_{j,t} A_t K_{j,t}^{\alpha-1} L_{j,t}^{\alpha-1} = 0 \quad (1.20)$$

Bearing in mind that $\mu_{j,t} = MC_{j,t}$ (Marginal Cost) ,the two equations below become the demand of a wholesale firm j for labor and capital equations, respectively:

$$L_{j,t} = (1-\alpha) MC_{j,t} \frac{Y_{j,t}}{W_t} \quad (1.21)$$

$$K_{j,t} = \alpha\, MC_{j,t} \frac{Y_{j,t}}{R_t} \quad (1.22)$$

Total cost is represented by:

$$TC_{j,t} = \frac{Y_{j,t}}{A_t} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} \quad (1.23)$$

And the marginal cost derived from the total cost:

$$MC_{j,t} = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t}{\alpha}\right)^{\alpha} \quad (1.24)$$

Secondly, the wholesaler defines the price of its goods using Calvo's rule (1983). In fact, following Celso (2016), the firm has not the total freedom to adjust its price each period. In terms of probabilities, the wholesale firm has a fixed $1-\theta$ probability of defining its prices and a θ probability of keeping prices fixed for a period, θ^2 for two periods and so on. So, Calvo's rule states that for a given period there is a fraction $0 < \theta < 1$ of firms selected randomly are allowed to define the prices in each period. The θ fraction, presents the rest of the firms that will maintain their prices rigid following one of the three following stickiness rules:

- Maintain the previous period's price

$$P_{j,t} = P_{j,t-1}$$

- Update the price using the steady state gross inflation rate (π_{ss})

$$P_{j,t} = \pi_{ss} P_{j,t-1}$$

- Update the price using the previous period's gross inflation rate (π_{t-1})

$$P_{j,t} = \pi_{t-1} P_{j,t-1}$$

The first rule is intended to be used to determine price stickiness for Tunisia; $P_{j,t} = P_{j,t-1}$.

Hence, the problem for a wholesaler to adjust the price of its goods is:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left(P^*_{j,t} Y_{j,t+1} - CT_{j,t+i}\right) \quad (1.25)$$

Substituting equation (1.13) in equation (1.25):

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[P^*_{j,t} Y_{t+i} \left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\psi} - Y_{t+i} \left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\psi} MC_{j,t+i}\right] \quad (1.26)$$

The first order condition is:

$$E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[(1-\psi) Y_{j,t+i} + \psi \frac{Y_{j,t+i}}{P^*_{j,t}} MC_{j,t+i}\right]$$

The wholesaler's price is:

$$P^*_{j,t} = \left(\frac{\psi}{\psi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i} \quad (1.27)$$

For all the 1-θ firms that set their prices, $P^*_{j,t}$ is the same. Furthermore, all the wholesale firms that fix their prices have the sale markup on the same marginal cost.

By combining markup rule equation and the evidence that the firms (both, ones that fix their prices and others that are supposed to defines theirs' subject to rigidity) have the same function, the general price level is described by the equation (1.28):

$$P_t^{1-\psi} = \int_0^{\theta} P_{t-1}^{1-\psi} \, dj + \int_0^{1} P_t^{*1-\psi} \, dj$$

Leading to:

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi}\right]^{\frac{1}{1-\psi}} \quad (1.28)$$

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The model's equilibrium condition

The economy's model consists of different equations cited in the section above that presents households' attitude, as well as firms' decision of their quantity of production with the optimal technology in addition to factors of production and prices settled by retailers and wholesalers. The structure of the model includes also the equilibrium condition in the goods market and gross inflation rate.

The equilibrium condition is defined by $Y_t = C_t + I_t + G_t + X_t - M_t$, with G_t,X_t and M_t are the government spending, exports and imports respectively.

The government spending follows first order autoregressive process as in Romer (2011):

$$log\ G_t = (1-\rho_G) \log G_{ss} + \rho_G\ log\ G_{t-1} + \epsilon_{G,t} \qquad (1.29)$$

Concerning the exports and imports process, we inspire from Romer (2011) and Joao and Madeira (2013) to set them respectively as:

$$logX_t = (1-Tx)\ log\ X_{ss} + Tx \log X_{t-1}\, \epsilon_{x,t} \qquad (1.30)$$

$$logM_t = (1-Tim)\ log\ M_{ss} + Tim \log M_{t-1} + \epsilon_{m,t} \qquad (1.31)$$

Where Tx is the ratio of the annual growth rate of exports to GDP and Tim is the ratio of the annual growth rate of imports to GDP.

Table 1-1 : model structure.

Equation	Definition
$C_t^{\sigma}\ L_t^{\varphi} = \frac{W_t}{P_t}$	(Labor supply)
$\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$	(Euler equation)
$K_{t+1} = (1 - \delta)\ K_t + I_t$	(Law of motion of capital)

$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ (Production function)

$K_t = \alpha\, MC_t \frac{Y_t}{R_t}$ (Demand for capital)

$L_t = (1-\alpha) MC_t \frac{Y_t}{W_t}$ (Demand for labor)

$P_j^* = \left(\frac{\psi}{\psi - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i\, MC_{t+i}$ (Optimal price level)

$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi}\right]^{\frac{1}{1-\psi}}$ (General price level)

$\pi_t = \frac{P_t}{P_{t-1}}$ (Gross inflation rate)

$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$ (Marginal cost)

$log\ G_t = (1 - \rho_G) \log G_{ss} + \rho_G\ log\ G_{t-1} + \epsilon_{G,t}$ (Government expenditure process)

$logX_t = (1 - Tx)\ log\ X_{ss} + Tx \log X_{t-1} + \epsilon_{x,t}$ (Exports)

$logM_t = (1 - Tim)\ log\ M_{ss} + Tim \log M_{t-1} + \epsilon_{m,t}$ (Imports)

$Y_t = C_t + I_t + G_t + X_t - M_t$ (Equilibrium condition)

$\text{Log } A_t = (1-\rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t$ (Productivity shock)

The steady state

An important step to resolve the system of equations is to define the steady state values. A model is in steady state when it exist a value for the variables that is maintained over time; an endogenous variable x_t is at the steady state in each t, if $E_t\ x_{t+1} = x_t = x_{t-1} = x_{ss}$ .It will be considered also that $E(\epsilon_t) = 0$ and $A_{ss} = 1$.

More details about the steady state steps are presented in Appendix C-1 and below there is the presentation of the steady state summarized:

$$A_{ss}=1$$

$$P_{ss}=1$$

$$C_{ss} = \frac{1}{Y_{ss}^{\frac{\varphi}{\sigma}}} \left[\frac{W_{ss}}{P_{ss}} \left(\frac{W_{ss}}{(1-\alpha) MC_{ss}} \right)^{\varphi} \right]^{\frac{1}{\sigma}}$$

$$I_{ss} = \left(\frac{\delta \alpha MC_{ss}}{R_{ss}} \right) Y_{ss}$$

$$W_{ss} = (1-\alpha) MC_{ss}{}^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_{ss}} \right)^{\frac{\alpha}{1-\alpha}}$$

$$MC_{ss} = \left(\frac{\psi}{\psi - 1} \right) (1 - \beta\theta) P_{ss}$$

$$R_{ss} = P_{ss} \left[\left(\frac{1}{\beta} \right) - (1-\delta) \right]$$

$$L_{ss} = (1-\alpha) MC_{ss} \frac{Y_{ss}}{\frac{W_{ss}}{P_{ss}}}$$

$$K_{ss} = \alpha\, MC_{ss} \frac{Y_{ss}}{\frac{R_{ss}}{P_{ss}}}$$

$$Y_{ss} = \left(\frac{R_{ss}}{R_{ss} - \delta \alpha MC_{ss}} \right)^{\frac{\sigma}{\sigma+\varphi}} \left[\frac{W_{ss}}{P_{ss}} \left(\frac{W_{ss}}{(1-\alpha) MC_{ss}} \right)^{\varphi} \right]^{\frac{1}{\sigma+\varphi}}$$

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This section deals with the necessary equations characterizing the equilibrium of the system. The next step is to log-linearize the model.

Log-linearization with Taylor expansion series

The above-mentioned model is nonlinear and necessitates complicated mathematical calculations. Following Zietz's work (2006), it is essential to compute the model's log-linear approximation around the steady state. According to the author, log-linearization, which is based on first-order Taylor approximations, helps in reducing mathematical complexity in systems of numerically specified equations that must be solved simultaneously.

The approach turns nonlinear equations into linear equations in terms of log-deviations of steady-state values from the related variables so that it will be easier to solve.

We refer to the linear approximation as the first-order Taylor approximation to f about $x = x_{ss}$ defined as: $f(x) = f(x_{ss}) + f'(x)(x - x_{ss})$

The log-linearization of each equation is presented in details in appendix C-2 and the structure of the log-linearized model is as follows:

Table 1-2: Structure of the log-linear model

Equation	Definition
$\sigma\tilde{C}_t + \varphi\tilde{L}_t = \tilde{W}_t - \tilde{P}_t$	(Labor supply)
$\frac{\sigma}{\beta}\left[E_t\tilde{C}_{t+1} - \tilde{C}_t\right] = \frac{R_{ss}}{P_{ss}}\left[E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1}\right]$	(Euler equation)
$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \frac{I_{ss}}{K_{ss}}\tilde{I}_t$	(Law of motion of capital)
$\tilde{Y}_t = \tilde{A}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{L}_t$	(Production function)
$\tilde{K}_t = \widetilde{MC}_t + \tilde{Y}_t - \tilde{R}_t$	(Demand for capital)
$\tilde{L}_t = \widetilde{MC}_t + \tilde{Y}_t - \tilde{W}_t$	(Demand for Labor)

$$\widetilde{MC}_t = (1-\alpha)\widetilde{W}_t + \alpha\widetilde{R}_t - \widetilde{A}_t$$ (Marginal cost)

$$\widetilde{\pi}_t = \beta E_t\widetilde{\pi}_{t+1} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right](\widetilde{MC}_t - \widetilde{P}_t)$$ (Phillips equation)

$$\widetilde{\pi}_t = \widetilde{P}_t - \widetilde{P}_{t-1}$$ (Gross inflation rate)

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$$ (Government process)

$$\widetilde{X_t} = Tx * \tilde{X}_{t-1} + \epsilon_{x,t}$$ (Exports)

$$\widetilde{M}_t = Tim * \widetilde{M}_{t-1} + \epsilon_{m,t}$$ (Imports)

$$\widetilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \tilde{X}_t - \widetilde{M}_t$$ (Equilibrium condition)

$$\widetilde{A}_t = \rho_A \widetilde{A}_{t-1} + \epsilon_t$$ (Productivity shock)

Calibration

Calibration is a process that consists of finding numerical values for the model's parameters in such a way that the steady state equilibrium corresponds with the stylized facts of the Tunisian economy. The main idea of this approach is to first establish, at the first stage, the economy's stationarity before exposing it to exogenous shocks.

In this work, only the estimation of imports and exports parameters has been carried out. Otherwise, we proceeded to implement the calibration of the parameters of the model from various review of literature in similar fields. First, following anterior Tunisian works (Ben Aissa & Rebei (2012), Jouini & Rebei (2014), Abdelli & Belhadj (2015), Ben Romdhane (2020)). Then the calibration generally accepted by the economic literature (Romer (2011), Celso (2016) and Othieno & Biekpe (2018)).

The discount rate β is set to 0.985 (Jouini & Rebei, 2014), which coincides with a real annual interest rate of 6%[5]. Following Ben Aissa & Rebei (2012), the depreciation rate δ is set at 0.0241. According to Abdelli & Belhadj (2015), the price stickiness parameter θ is set at 0.75, the elasticity of substitution between intermediate goods ψ is set at 8, the elasticity of output with respect to capital α is 0.35, the standard deviation of productivity σ_A is set at 0.01, the autoregressive parameter of productivity ρ_A equals to 0.8 and the Elasticity of output with respect to capital α, is set to 0.35.

For the parameters in the government's equation, we follow Romer (2011) to put sg and ρ_G as 0.2 and 0.95 respectively.

For the marginal disutility with respect to labor supply parameter φ, we followed Ben Hassine & Rebei (2019) and Ben Romdhane (2020), to set it at 1.φ is simply the inverse of the Frisch elasticity of labor supply, which is the trade-off between labor and leisure. It is the reduction of 1 hour of leisure while working one hour more.

Analysis of shocks

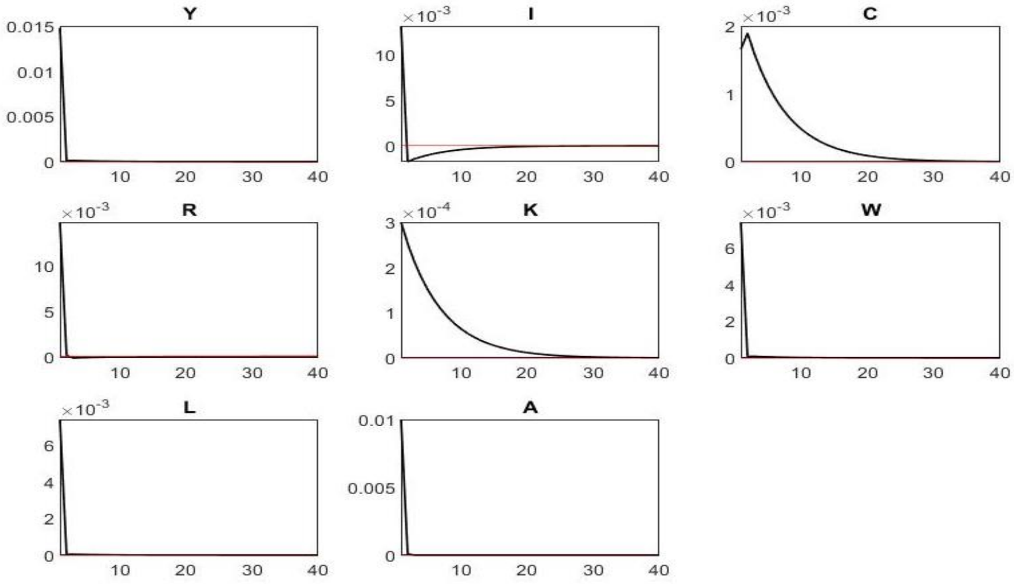
Impulse Response Functions (IRFs), generated by Matlab R2020a and Dynare 4.6.3, are used to demonstrate the behavior of the model and the fit to the data. They also demonstrate the reactions of the macroeconomic variables to one-standard deviation shocks.

[5] With an annual interest rate of 6%, the discount factor is calculated as follows: β = 1 / (1+0.06/4).

All impulse responses are reported as percentage deviations

Figure 1-4: Effects of a productivity shock

from the model's steady state.



Source: Matlab-Dynare simulation results.
Note: The abscissa line denotes the quarters and the ordinate line denotes the percentage-point deviation from the steady state.

Figure 1-4 shows the effects of a positive disruption of the total productivity of the factors of production. The first evidence of this shock is the increase of the marginal productivity of labor capital which affects the prices of the factors of production (W and R). With higher wages, the household increases its acquisition of goods; consumption and Investment (at the short time).

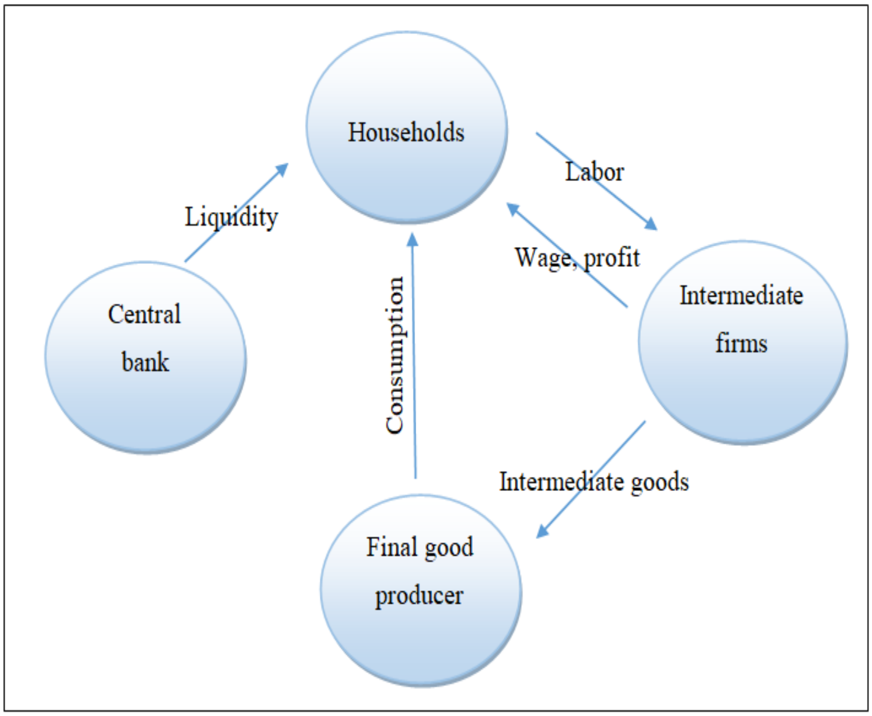
However, as it is shown in the figure, the households consume more than saving (C increases and I decreases sharply). Demand for inputs (L and K) increase, but as wages decrease over time, households seek more leisure which explains the decrease of the number of hours worked.

Impact of monetization on Tunisian economy: Evaluation with a New Keynesian DSGE model with monetary authority

The standard DSGE model with money

Up to this point, only three agents have been considered: households, firms and government. In this section, an important agent will be added which is the monetary authority (see figure 1-5). Thus, this section examines the role of the role of monetary authority.

Figure 1-5: Flowchart of the economy



Source: Author's elaboration

This section seeks to present different equations of the simple DSGE model (households, retailers, wholesalers and the monetary authority). The model is inspired from Romer (2011), Junior (2016) and Sims (2017), which will be modeled to Tunisia.

The model, as like as the previous section, takes in consideration price stickiness and monopolistic competition. The same equations remain the same (for the firms and the government) except for the household behavior.

1.1.2 Households

Our SOE is formed by a unitary set of households indexed by j ϵ [0,1]. The representative household consumes, supplies labor, accumulates bonds, holds shares in firms, and accumulates money. His problem is to maximize the following welfare function additively separable into Consumption (C), Labor (L) and real balances ($\frac{M}{P}$):

$$\max_{C_{j,t},\, L_{j,t}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} + \kappa \ln\left(\frac{M_{j,t}}{P_{j,t}}\right) \right) \quad (1.32)$$

It is to be expected that a rise in consumption and real balances raises utility and a rise in labor hours brings disutility.

Where E_t is the expectations operator, β is the intertemporal discount factor, C is goods consumption, L is the number of hours worked, σ is the relative risk aversion coefficient, φ is the marginal disutility in respect of labor supply.

Sims (2017) supposed that the money supply follows an AR (1) in the growth rate, where $\Delta\log M_t = \log M_t - \log M_{t-1}$:

$$\Delta\log \mathrm{M_t} = (1-\rho_m)\pi + \rho_m \Delta\log \mathrm{M_{t-1}} + \varepsilon_{m,t} \quad (1.33)$$

Households maximize their welfare function subject to intertemporal budget constraint, indicating the resources available and how they are allocated. The intertemporal budget constraint of the households can be written as follows:

$$P_t C_{j,t} + B_{j,t+1} + M_{j,t} - M_{j,t-1} \leq W_t L_{j,t} + \Pi_t + (1+i_{t-1}) B_{j,t} \quad (1.34)$$

Where P is the general price level, W is the level of wages, Π is the firms' profits, B_t is the stock of nominal bonds a household enters the period with, i_{t-1} is the nominal interest rate. The household also enters the period with a stock of money $M_{j,t-1}$. M_t is set by a central bank.

Another important component is the capital accumulation equation which shows how the capital stock changes over years:

$$K_{j,t+1} = (1-\delta) K_{j,t} + I_{j,t} \quad (1.35)$$

Equations (1.32), (1.34) and (1.35) establish the households' problem. The following Lagrangian will be used in order to solve it:

$$\mathcal{L} = \mathrm{E_t} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} + \kappa \, ln\left(\frac{M_{j,t}}{P_t}\right) \right) + \lambda_{j,t} \left(W_t L_{j,t} + \Pi_t + (1 + i_{t-1}) B_{j,t} - P_t C_{j,t} - B_{j,t+1} - M_{j,t} + M_{j,t-1} \right) \right)$$

(1.36)

When deriving the Lagrange to consumption, to labor, to bonds and to money, first-order conditions (FOC) are obtained:

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0$$

(1.37)

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -L_{j,t}^{\varphi} + \lambda_{j,t} W_t = 0$$

(1.38)

$$\frac{\partial \mathcal{L}}{\partial B_{j,t+1}} = -\lambda_{j,t} - \beta \, \mathrm{E_t} \, \lambda_{j,t+1} (1 + i_{t-1}) = 0$$

(1.39)

$$\frac{\partial \mathcal{L}}{\partial M_{j,t}} = \kappa \left(\frac{1}{M_t}\right) - \lambda_{j,t} + \beta \, \mathrm{E_t} \, \lambda_{j,t+1} = 0$$

(1.40)

Solving for λ_t equations (1.37) and (1.38), the household's labor supply equation is obtained:

$$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$$

(1.41)

Euler equation is found after determining from equation (1.37) $\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}$ and $\lambda_{j,t+1} = \frac{C_{j,t+1}^{-\sigma}}{P_{j,t+1}}$, then using these results in equation (1.39):

$$C_{j,t}^{-\sigma} = \beta E_t \, C_{j,t+1}^{-\sigma} (1 + i_t) \frac{P_t}{P_{t+1}}$$

(1.42)

Thus, a surge in interest rate expectations will make the present consumption more expensive.

Note that β = 1 and $\frac{P_t}{P_{t-1}} - 1$ is the inflation rate π_t. The Euler equation can be re-written:

$$C_{j,t}^{-\sigma} = E_t \, C_{j,t+1}^{-\sigma} (1 + i_t)(1 + \pi_{t+1})^{-1}$$

The money demand equation is determined by replacing the values of $\lambda_{j,t}$ and $\lambda_{j,t+1}$ from equations (1.36) and (1.38) respectively, in equation (1.40):

$$\kappa\left(\frac{M_{j,t}}{P_t}\right)^{-1} = \frac{i_t}{1+i_t}\ C_{j,t}^{-\sigma}$$

After simplification, the money demand equation becomes:

$$\frac{M_{j,t}}{P_t} = \kappa\ \frac{1+i_t}{i_t}\ C_{j,t}^{\sigma}$$

The demand for money equation is already written in terms of real balances, $m_t = \frac{M_t}{P_t}$:

$$m_t = \kappa\ \frac{1+i_t}{i_t}\ C_{j,t}^{\sigma}$$

(1.43)

Monetary authority

Monetary authority is considered as the last agent in the model. The main objective of the CBT is to ensure price stability contributing to financial stability in a way to boost growth and minimize unemployment.

Monetary authority focuses on stabilizing prices and economic growth using the Taylor rule. Following Celso (2016) who used simple Taylor rule as a function of past interest rate, inflation and output:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B}\right)^{\gamma_R}\ \left(\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y}\right)^{1-\gamma_R}\ S_t^m$$

(1.44)

Where γ_Y is the sensitivity of the basic interest rate in relation to the product, γ_π the sensitivity of the basic interest rate in relation to the inflation rate, γ_R is the smoothing parameter. S_t^m is the monetary shock presented by:

$$logS_t^m = (1-\rho_m)logS_{ss}^m + \rho_m logS_{t-1}^m + \varepsilon_{m,t} \qquad (1.45)$$

The equilibrium of the economy is characterized by the first order conditions of the household, the firms and the Taylor rule.

Log-linearization with Taylor series expansion

The method developed by Sims (2017) and Zietz (2016) continues to be used as a tool for the log-linearization of the model. The results of the section below will be used.

In this section, the new element is the log-linearization of the money demand function and Euler equation. Taylor rule and the monetary shock will be log-linearized also (Appendix C-3).

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The log-linearized model is as follow:

Equation	Definition
$\sigma\tilde{C}_t + \varphi\tilde{L}_t = \widetilde{W}_t - \widetilde{P}_t$	(Labor supply)
$\tilde{C}_t - E\tilde{C}_{t+1} = -\frac{1}{\sigma}(\tilde{\imath}_t - E\tilde{\pi}_{t+1})$	(Euler equation)
$\widetilde{m_t} = \left(1 - \frac{\beta}{1-\beta}\right)\widetilde{\imath_t} + \sigma\tilde{C}_t$	(Money demand)
$\widetilde{\Delta m}_t = -\tilde{\pi}_t + \rho_m\tilde{\pi}_{t-1} + \rho_m\Delta\widetilde{m}_{t-1} + \varepsilon_{m,t}$	(real balance growth)
$\widetilde{\Delta m}_t = \widetilde{m}_t - \widetilde{m}_{t-1}$	(real balance growth definition)
$\widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \frac{I_{ss}}{K_{ss}}\tilde{I}_t$	(Capital accumulation)
$\widetilde{Y}_t = \widetilde{A}_t + \alpha\widetilde{K}_t + (1-\alpha)\tilde{L}_t$	(Production function)
$\widetilde{K}_t = \widetilde{MC}_t + \widetilde{Y}_t - \widetilde{R}_t$	(Demand for capital)
$\tilde{L}_t = \widetilde{MC}_t + \widetilde{Y}_t - \widetilde{W}_t$	(Demand for Labor)
$\widetilde{MC}_t = (1-\alpha)\widetilde{W}_t + \alpha\widetilde{R}_t - \widetilde{A}_t$	(Marginal cost)
$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right](\widetilde{MC}_t - \widetilde{P}_t)$	(Phillips curve equation)
$\tilde{\pi}_t = \widetilde{P}_t - \widetilde{P}_{t-1}$	(Gross inflation rate)

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$$ (Government spending process)

$$\widetilde{X_t} = Tx * \tilde{X}_{t-1} + \epsilon_{x,t}$$ (Exports process)

$$\widetilde{M}_t = Tim * \widetilde{M}_{t-1} + \epsilon_{m,t}$$ (Imports process)

$$\widetilde{R_t} = \gamma_R \tilde{R}_{t-1} + (1+\gamma_R)\left(\gamma_\pi \tilde{\pi}_t + \gamma_Y \tilde{Y}_t\right) + \tilde{S}_t^m$$ (Taylor rule)

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + \tilde{X}_t - \widetilde{M}_t$$ (Equilibrium condition)

$$\widetilde{A}_t = \rho_A \widetilde{A}_{t-1} + \epsilon_t$$ (Productivity shock)

$$\tilde{S}_t^m = \rho_m \tilde{S}_{t-1}^m + \epsilon_{m,t}$$ (Monetary shock)

Calibration

The calibration belongs to the parameters in the added equations. Like in Abdelli & Belhadj (2015) and Celso (2016), the Autoregressive parameter of money ρ_m and the Standard deviation of money σ_m are put to 0.9 and 0.005 respectively. The trade-off between money demand and consumption κ is set to 1 as in Sims (2017).

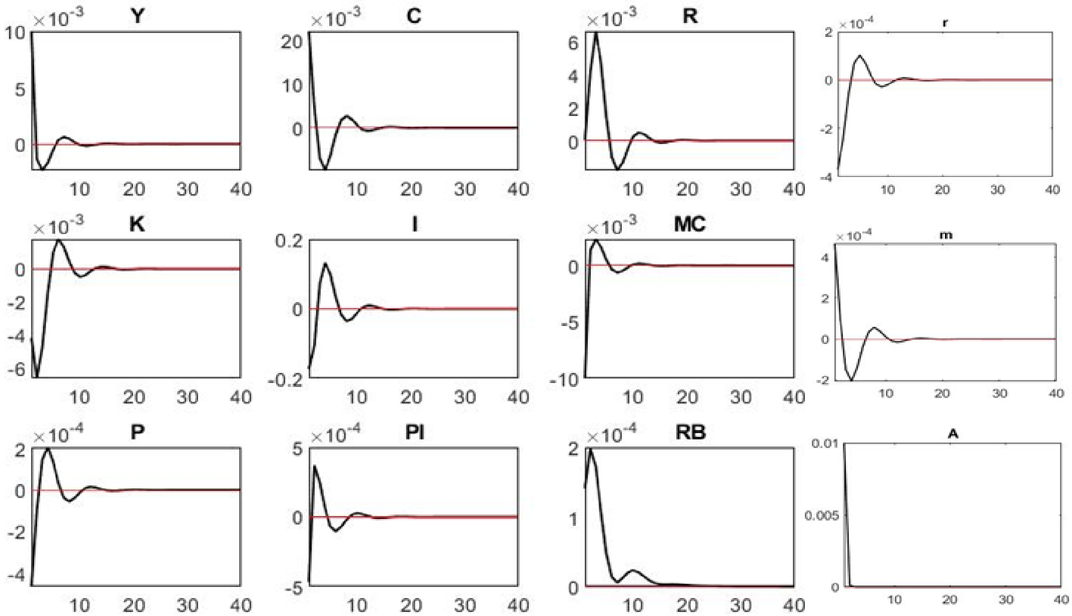
The parameters describing monetary policy are based in Taylor rule. For the calibrated values which are quite standard in the literature, this work refers to the values of Celso (2016) and Ben Romdhane (2020) to fix the smoothing parameter at 0.8. γ_Y and γ_π are 0.15 and 1.7 respectively.

Analysis of shocks

- Impulse responses following Productivity shock

Figure 1-6 presents the supply shock for the model with the monetary authority. When productivity is high, the representative intermediate goods-producing firms may create more items at a cheaper cost (Abdelli & Belhadj (2015)). Therefore, it can be deduced that a supply shock has a positive effect on production. The rise in the productivity and an improvement in the production technology lead to an increase in the output (Y) and the consumption (C) which is similar to the findings of Adnen & Frikha (2011) and Abdelli & Belhadj (2015).

Figure 1-6: Effects of productivity shock (model with monetary authority)



Source: Matlab-Dynare simulation results.
Note: The abscissa line denotes the quarters and the ordinate line denotes the percentage-point deviation from the steady state.

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This shock decreases the inflation (PI) in the short term and increases real balances; this result is similar to Kremer et al. (2013) and Castelnuovo (2012). The marginal cost (MC) and the interest rate decrease also due to the rise in the output.

Finally, the Investment (I) drops, this result is consistent with the findings of Abdelli & Belhadj (2015). This decrease can be explained by the depreciation of the real interest rate (r) (Jouini and Rebei, 2013).

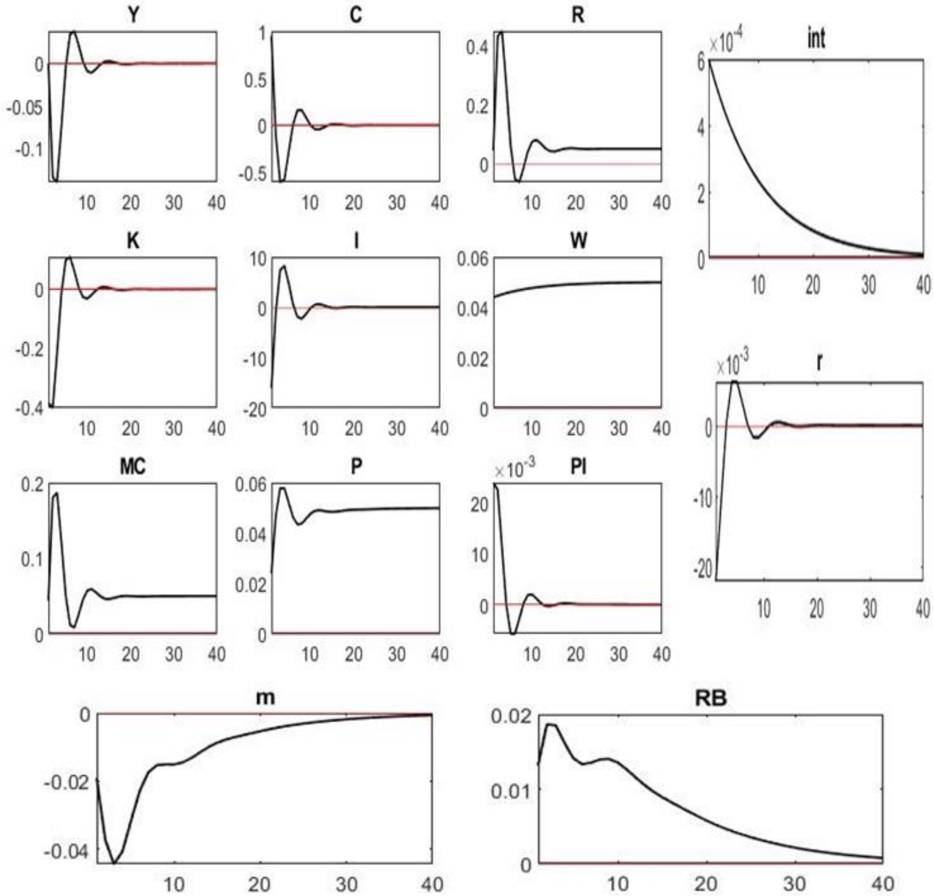
- Impulse responses following a money demand shock

The figure 1-7 shows the IRFs following monetary shock which assumes an expansionary monetary policy.

It can be observed that there is a persistent decline over the output and the investment like as in (Dupless et al. (2008), Drissi & Gaaloul (2013) and Kemp et al. (2020)).

The inflation (PI) and the marginal costs (MC) raise. As a consequence, wages (W) jump but never get back to the steady state. Sims (2017) claims that this increase is essential in order to get workers to work more.

Figure 1-7: Effects of a monetary shock (model with monetary authority)



Source: Matlab-Dynare simulation results.
Note: The abscissa line denotes the quarters and the ordinate line denotes the percentage-point deviation from the steady state.

There is also a less significant decrease in the money demand (m) coupled with a slight drop in investment which will be recovered from the 5th period both. This could be explained by the household's reticence to cash investments. With sticky prices the inflation can't increase sufficiently, so money demand rises and therefore so does the output (Y) too. Higher inflation lowers real interest rate, which stimulates results in the output increase (Sims, 2017).

Conclusion and discussion

This paper provides a small open economy DSGE model to evaluate the impact of monetization on the Tunisian economy. The model is framed in the NK tradition where prices are sticky and firms are not assumed to adjust prices frequently. The key structural parameters of the model are extracted from recent Tunisian and specialized literature.

The empirical exercise showed that the different shocks have an immense role in explaining the movements in key economic variables. The findings reveal that the technology shock raises output and consumption and reduces inflation, which is a common result in different works. Overall, the productivity shock has a positive effect on the macroeconomic variables.

On the other hand, the expansionist monetary shock affects negatively the capital and the investment which by its turn pushes the output to decrease. The drop in investment is explained by the households' reticence to cash investments. Prices jump significantly without coming back to the steady state accelerating inflation. Wages hike without achieving the steady state in order to encourage Tunisian workers to work more.

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Appendices

Appendix C-1: the steady state model detailed

An important step to resolve the system of equations is to define the steady state values. A model is in steady state when it exist a value for the variables that is maintained over time; an endogenous variable xt is at the steady state in each t, if Et xt+1= xt= xt-1= xss .It will be considered also that E (ϵt) = 0 and Ass= 1.

The model in steady state is as follows:

Households

$$C_{ss}^{\sigma} L_{ss}^{\varphi} = \frac{W_{ss}}{P_{ss}} \quad \text{(C1.1)}$$

$$1 = \beta\left[1 - \delta + \left(\frac{R_{ss}}{P_{ss}}\right)\right] \quad \text{(C1.2)}$$

$$\delta \; K_{ss} = I_{ss} \quad \text{(C1.3)}$$

Firms

$$K_{ss} = \alpha \, MC_{ss} \; \frac{Y_{ss}}{\frac{R_{ss}}{P_{ss}}} \quad \text{(C1.4)}$$

$$L_{ss} = (1-\alpha) MC_{ss} \; \frac{Y_{ss}}{\frac{W_{ss}}{P_{ss}}} \quad \text{(C1.5)}$$

$$Y_{ss} = K_{ss}^{\alpha} L_{ss}^{1-\alpha} \quad \text{(C1.6)}$$

$$MC_{ss} = \left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_{ss}}{\alpha}\right)^{\alpha} \quad \text{(C1.7)}$$

$$P_{ss} = \left(\frac{\psi}{\psi-1}\right)\left(\frac{1}{1-\beta\theta}\right) MC_{ss} \quad \text{(C1.8)}$$

Equilibrium condition

$$Y_{ss} = C_{ss} + I_{ss} + G_{ss} + X_{ss} - M_{ss} \quad \text{(C1.9)}$$

The values of general price level Pss, capital remuneration Rss, marginal cost MCss and the wage level Wss are the first to be determined by taking into consideration Walras' Law.

Walras' Law states that for any price vector p, the demand excess value is identically to zero.

The demand excess value is defined as:

$$pz(p) = p\left[\sum_{i=0}^{n} x_i(p, p\, w_i) - \sum_{i=0}^{n} w_i\right]$$

$$= \sum_{i=1}^{n} [p x_i(p, p\, w_i) - p\, w_i] = 0$$

Walras' Law affirms that the sum of excess demand must be equal to zero for all prices if each individual satisfies his budget constraint. This law considers that if k-1 markets are at equilibrium then the k^th market will be also. Shortly, taking Walras' Law in consideration, P_{ss}=1.

To find R_{ss}, equation (C1.2) is used:

$$R_{ss} = P_{ss}\left[\left(\frac{1}{\beta}\right) - (1 - \delta)\right] \quad \text{(C1.10)}$$

From equation (C1.8):

$$MC_{ss} = \left(\frac{\psi}{\psi - 1}\right)(1\beta\theta\ )P_{ss} \quad \text{(C1.11)}$$

It remains thus to find the value of the wage level at the steady state from equation (C1.7):

$$W_{ss} = (1 - \alpha) MC_{ss}{}^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_{ss}}\right)^{\frac{\alpha}{1-\alpha}} \quad \text{(C1.12)}$$

After determining the prices it is essential to satisfy the equilibrium condition by finding out the variables that make up aggregate demand (C_{ss} and I_{ss}).

In order to determine the equilibrium condition, formed by market adjustment proposition; given k markets, if, in k-1 markets, demand is equal to supply and $p_k > 0$, then supply must equal supply in the k^th market. Therefore the meeting point between supplies and demands of the production inputs must be found.
At first, equation (C1.4) must be replaced in equation (C1.3), solving for I_{ss}:

$$I_{ss} = \left(\frac{\delta\alpha MC_{ss}}{R_{ss}}\right)\ Y_{ss} \quad \text{(C1.13)}$$

Consequently, equation (C1.5) must be replaced in equation (C1.4) to find the value of C_{ss}:

$$C_{ss}^{\sigma}\left[(1-\alpha)MC_{ss}\ \frac{Y_{ss}}{\frac{W_{ss}}{P_{ss}}}\right]^{\varphi}=\frac{W_{ss}}{P_{ss}}$$

$$C_{ss}=\frac{1}{Y_{ss}^{\frac{\varphi}{\sigma}}}\left[\frac{W_{ss}}{P_{ss}}\left(\frac{W_{ss}}{(1-\alpha)MC_{ss}}\right)^{\varphi}\right]^{\frac{1}{\sigma}}$$

(C1.14)

At this point, the value of Y_{ss} must be found, after substituting equation (C1.13) and equation (C1.14) in equation (C1.10), Y_{ss} equals to:

$$Y_{ss}=\left(\frac{R_{ss}}{R_{ss}-\delta\alpha MC_{ss}}\right)^{\frac{\sigma}{\sigma+\varphi}}\left[\frac{W_{ss}}{P_{ss}}\left(\frac{W_{ss}}{(1-\alpha)MC_{ss}}\right)^{\varphi}\right]^{\frac{1}{\sigma+\varphi}} \quad \text{(C1.15)}$$

Appendix C-2: Log linearization of the model without money

Assume x_{ss} denotes the steady state value of variable x_t. Next, define the log-deviation of variable x_t from its steady state x_{ss} as: $\tilde{x}_t = \ln\ x_t - \ln x_{ss}$. The author then suggests the following problem-solving tools:

$$x_t = x_{ss}(1+\tilde{x}_t)$$

(C2.1)

$$\tilde{x}_t = \frac{x_t - x_{ss}}{x_{ss}}$$

(C2.2)

$$e^{\tilde{x}_t} = 1 + \tilde{x}_t$$

(C2.3)

The steps for log-linearizing as proposed by Sims (2011) are:

1. Take logs.
2. Do a first order Taylor series expansion about the steady state.
3. Simplify to percentage deviations from steady state.

- **Labor supply**

Starting with the labor supply equation $C_t^{\sigma}\ L_t^{\varphi} = \frac{W_t}{P_t}$

First take logs: $\sigma \ln C_t + \varphi \ln L_t = \ln W_t - \ln P_t$

Now do the Taylor series expansion about the steady state values:

$$\sigma\, ln\, C_{ss} + \frac{\sigma}{C_{ss}}(C_t - C_{ss}) + \varphi\, ln\, L_{ss} + \frac{\varphi}{L_{ss}}(L_t - L_{ss})$$
$$= ln\, W_{ss} + \frac{\varphi}{W_{ss}}(W_t - W_{ss}) - ln\, P_t - \frac{\varphi}{P_{ss}}(P_t - P_{ss})$$

Since at the steady state $\sigma \ln C_{ss} + \varphi \ln L_{ss} = \ln W_{ss} - \ln P_{ss}$, then:

$$\sigma \tilde{C}_t + \varphi \tilde{L}_t = \widetilde{W}_t - \tilde{P}_t$$

(C2.4)

- Euler equation

Rewriting Euler equation,

$$\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$

By replacing $X_t = X_{ss} e^{\tilde{X}_t}$

$$\frac{1}{\beta}\left[1 + \sigma(E_t \tilde{C}_{t+1} - \tilde{C}_t)\right] = (1-\delta) + \frac{R_{ss}}{P_{ss}}[1 + (E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1})]$$

Given that the steady state $\frac{1}{\beta} = \frac{R_{ss}}{P_{ss}} + (1-\delta)$:

$$\frac{\sigma}{\beta}\left[E_t \tilde{C}_{t+1} - \tilde{C}_t\right] = \frac{R_{ss}}{P_{ss}}[E_t(\tilde{R}_{t+1} - \tilde{P}_{t+1}] \qquad \text{(C2.5)}$$

- Production function

Using the same procedure as before:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$\ln Y_t = \ln A_t + \alpha \ln K_t + (1-\alpha) \ln L_t$$

$$\ln Y_{ss} + \frac{1}{Y_{ss}}(Y_t - Y_{ss}) =$$

$$\ln A_{ss} \frac{1}{A_{ss}}(A_t - A_{ss}) + \ln K_{ss} + \frac{\alpha}{K_{ss}}(K_t - K_{ss}) + \ln L_{ss} + \frac{(1-\alpha)}{L_{ss}}(L_t - L_{ss})$$

As above, note that $lnY_{ss} = \ln A_{ss} + \alpha \ln K_{ss} + (1-\alpha) \ln L_{ss}$, so these terms cancel out:

$$\tilde{Y}_t = \tilde{A}_t + \alpha \widetilde{K}_t + (1-\alpha) \tilde{L}_t$$

(C2.6)

The same procedure of log-linearization is used for the next variables.

- Return on capital

Return on capital is:

$$K_t = \alpha\, MC_t \frac{Y_t}{P_t}$$

After taking logs in the previous equation and using a little algebra:

$$\widetilde{K}_t = \widetilde{MC}_t + \tilde{Y}_t - \tilde{R}_t$$

(C2.7)

- **Wage levels**

Demand for labor is: $L_t = (1-\alpha)MC_t \frac{Y_t}{W_t}$

After Taylor expansion is used:

$$ln\, L_{ss} + \frac{1}{L_{ss}}(L_t - L_{ss}) =$$

$$\ln MC_{ss} + \frac{1}{MC_{ss}}(MC_t - MC_{ss}) + \ln Y_{ss}\frac{1}{Y_{ss}}(Y_t - Y_{ss}) - \ln W_{ss} - \frac{1}{W_{ss}}(W_t - W_{ss})$$

Thus:

$$\tilde{L}_t = \widetilde{MC}_t + \tilde{Y}_t - \widetilde{W}_t$$

(C2.8)

- Capital accumulation

Following Sims (2017), the capital accumulation equation becomes with Taylor approximation at the steady state values:

$$\ln K_{ss} + \frac{1}{K_{ss}}(K_{t+1} - K_{ss}) =$$

$$\ln\big((1-\delta)K_{ss} + I_{ss}\big) + \frac{(1-\delta)}{(1-\delta)K_{ss} + I_{ss}}(K_t - K_{ss}) + \frac{1}{(1-\delta)K_{ss} + I_{ss}}(I_t - I_{ss})$$

Now simplify terms, noting that $\ln\big((1-\delta)K_{ss} + I_{ss}\big) = \ln K_{ss}$, so that terms cancel:

$$\frac{1}{K_{ss}}(K_{t+1} - K_{ss}) = \frac{(1-\delta)}{K_{ss}}(K_t - K_{ss}) + \frac{1}{K_{ss}}(I_t - I_{ss})$$

Now multiply and divide the right hand side by I_{ss}:

$$\frac{1}{K_{ss}}(K_{t+1} - K_{ss}) = \frac{(1-\delta)}{K_{ss}}(K_t - K_{ss}) + \frac{I_{ss}}{K_{ss}}\frac{(I_t - I_{ss})}{I_{ss}}$$

Capital accumulation becomes:

$$\widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \frac{I_{ss}}{K_{ss}}\tilde{I}_t \qquad \text{(C2.9)}$$

- **Government**

The process of government expenditure is:

$$log\ G_t = (1-\rho_G)\log G_{ss} + \rho_G\ log\ G_{t-1} + \epsilon_{G,t}$$

After transformations

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$$

(C2.10)

- **Exports**

From equation (C1.2), the log-linearized equation of exports becomes:

$$\widetilde{X_t} = Tx * \tilde{X}_{t-1} + \epsilon_{x,t}$$

(C2.11)

- **Imports**

The same procedure for the imports equation:

$$\widetilde{M}_t = Tim * \widetilde{M}_{t-1} + \epsilon_{m,t}$$

(C2.12)

- **Equilibrium condition**

Knowing that $Y_t = C_t + I_t + G_t + X_t - M_t$, Taylor expansion is as follows:

$$\begin{aligned} ln\,Y_{ss} + \frac{1}{Y_{ss}}(Y_t - Y_{ss}) = {} & \ln\ (C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}) \\ & + ln\frac{1}{C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}}(C_t - C_{ss}) \\ & + ln\frac{1}{C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}}(I_t - I_{ss}) \\ & + ln\frac{1}{C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}}(G_t - G_{ss}) \\ & + ln\frac{1}{C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}}(X_t - X_{ss}) \\ & - ln\frac{1}{C_{ss} + I_{ss} + \mathrm{G_{ss}} + \mathrm{X_{ss}} - \mathrm{M_{ss}}}(M_t - M_{ss}) \end{aligned}$$

With $G_{ss} = s_g Y_{ss}, X_{ss} = Tx * Y_{ss}\ and\ M_{ss} = Tim * Y_{ss}$ are the steady state government expenditure, exports and imports .

Then the equilibrium condition is:

$$\widetilde{Y}_t = \widetilde{C}_t + \tilde{I}_t + \tilde{G}_t + \tilde{X}_t - \widetilde{M}_t$$

(C2.13)

- **Technological shock**

The process of motion of productivity is:

Log A_t= (1-ρ_A) log A_{ss}+ ρ_A log A_{t-1}+ϵ_t

To convert to log deviations, one should replace the time sub-scripted variables per $x_t = e^{\ln x + \widetilde{x_t}} = e^{\ln x} e^{\widetilde{x_t}} = x e^{\widetilde{x_t}}$:

$$\ln \mathrm{A_{ss}}\, e^{\tilde{A}_t} = (1 - \rho_A)\ln \mathrm{A_{ss}} + \rho_A \ln \mathrm{A_{ss}} e^{\tilde{A}_{t-1}} + \epsilon_t$$

After transformations:

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_t$$

(C2.14)

- **Marginal cost**

The marginal cost needs to be log-linearized also. From equation (1.27):

$$MC_{ss}\left(1 + \widetilde{MC}_t\right) = \left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_{ss}}{\alpha}\right)^{\alpha} \left[1 - \tilde{A}_t + (1-\alpha)\widetilde{W}_t + \alpha \tilde{R}_t\right]$$

Therefore:

$$\widetilde{MC}_t = (1-\alpha)\widetilde{W}_t + \alpha \tilde{R}_t - \tilde{A}_t$$

(C2.15)

- **Philips Curve**

At first, the optimal price level's equation must be linearized, from equation (1.27) the linearized equation becomes:

$$\tilde{P}_t^* = (1 - \beta\theta)\, E_t \sum_{i=0}^{\infty} (\beta\theta)^i\, \widetilde{MC}_{t+i}$$

(C2.16)

Then, from equation (1.28), the final markup rule is linearized:

$$\tilde{P}_t = \theta \tilde{P}_{t-1} + (1-\theta)\widetilde{P*}_t$$

(C2.17)

By substituting equation (C2.16) in equation (C2.17):

$$\tilde{P}_t = \theta \tilde{P}_{t-1} + (1-\theta)(1 - \beta\theta) E_t \sum_{i=0}^{\infty} (\beta\theta)^i\, \widetilde{MC}_{t+i} \quad \text{(C2.18)}$$

At second, by a technique known as "quasi-differencing", the second element in the equation (C2.18) must be removed for the one and only cause that it contains an infinite sum of the future marginal cost. Both side of the equation (C2.18) must be multiplied by the lag operator (1- $\beta\theta L^{-1}$). When it is applied to a variable X_t it results in $L^{-1} X_t = X_{t+1}$.

Hence, multiplying equation (C2.18) by (1- $\beta\theta L^{-1}$):

$$\tilde{P}_t - \beta\theta E_t\tilde{P}_{t-1}$$
$$= \theta\tilde{P}_{t-1} + (1-\theta)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\,\widetilde{MC}_{t+i}$$
$$-\beta\theta\theta\tilde{P}_t - \beta\theta(1-\theta)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\,\widetilde{MC}_{t+1+i}$$

After some mathematical modifications, the New-Keynesian Phillips equation:

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right](\widetilde{MC}_t - \tilde{P}_t) \qquad \text{(C2.19)}$$

Appendix C-3: Log linearization of the model with money

- **Money demand function**

Money demand function is:

$$m_t = \kappa\,\frac{1+i_t}{i_t}\,C_{j,t}^{\sigma}$$

First take logs:

$$\ln m_t = \ln\kappa + \ln\frac{1+i_t}{i_t} + \sigma\ln C_t$$
$$\ln m_t = \ln\kappa + \ln(1+i_t) - \ln i_t + \sigma\ln C_t$$

Now do the first order Taylor series expansion:

$$\ln m_{ss} + \frac{1}{m_{ss}}(m_t - m_{ss}) =$$
$$\ln\kappa + \ln\ (1+i_{ss}) + \frac{1}{1+i_{ss}}(i_t - i_{ss}) - \ln i_{ss} - \frac{1}{i_{ss}}(i_t - i_{ss})$$
$$+\sigma\ln C_{ss} + \frac{\sigma}{C_{ss}}(C_t - C_{ss})$$

At the steady state: $\ln m_{ss} = \ln\kappa + \ln(1+i_{ss}) - \ln i_{ss} + \sigma\ln C_{ss}$, so these terms cancel out:

$$\frac{1}{m_{ss}}(m_t - m_{ss}) = \frac{1}{1+i_{ss}}(i_t - i_{ss}) - \frac{1}{i_{ss}}(i_t - i_{ss}) + \frac{\sigma}{C_{ss}}(C_t - C_{ss})$$

Since i_t is already a percent, it is common to leave it in absolute deviations. Hence, as in Sims (2011), we can define $\tilde{\imath}_t = i_t - i_{ss}$, while the "tilde" notation will be used as before for all variables such as consumption and money demand.

Furthermore, as the discount rate is sufficiently high (0.985), the term $\frac{1}{1+i_{ss}}$ can be approximated to 1, this will be good approximation. Then by simplifying:

$$\widetilde{m_t} = \widetilde{\iota_t}\left(1 - \frac{1}{i_{ss}}\right) + \sigma\widetilde{C_t}$$

Since $i_{ss} = \frac{1}{\beta} - 1$ then money demand function in logs is:

$$\widetilde{m_t} = \left(1 - \frac{\beta}{1-\beta}\right)\widetilde{\iota_t} + \sigma\tilde{C}_t \quad \text{(C3.1)}$$

- **Money growth**

Sims (2017) supposed that the money supply follows an AR (1) in the growth rate, where, in terms of real balances, $\Delta\, logm_t = logm_t - logm_{t-1}$:

$$\Delta\, logm_t = (1-\rho_m)\pi - \pi_t + \rho_m\, \Delta logm_{t-1} + \rho_m\pi_{t-1} + \varepsilon_{m,t}$$

After transformations:

$$\Delta\widetilde{m_t} = -\tilde{\pi}_t + \rho_m\tilde{\pi}_{t-1} + \rho_m\Delta\tilde{m}_{t-1} + \varepsilon_{m,t} \quad \text{(C3.2)}$$

$$\Delta\widetilde{m_t} = \tilde{m}_t - \tilde{m}_{t-1} \quad \text{(C3.3)}$$

- **Euler equation**

Rewriting Euler equation,

$$\frac{1}{\beta}\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = E_t(1+i_t)\left(\frac{1}{\pi_{t+1}}\right)$$

By taking logs and using Taylor series expansion:

$$-\sigma\, ln\, C_{ss} + \frac{\sigma}{C_{ss}}(C_t - C_{ss}) + \sigma ln\, C_{ss} + \frac{\sigma}{C_{ss}}E(C_{t+1} - C_{ss}) =$$

$$ln\beta + \ln(1+i_{ss}) + \frac{1}{1+i_{ss}}(i_t - i_{ss}) - ln(1+\pi_{ss}) - \frac{1}{1+\pi_{ss}}E(\pi_{t+1} - \pi_{ss})$$

Now simplify and do the "tilde" notation:

$$\tilde{C}_t - E\tilde{C}_{t+1} = -\frac{1}{\sigma}\left(\frac{1}{1+i_{ss}}\tilde{\imath}_t - \frac{1}{1+\pi_{ss}}E\tilde{\pi}_{t+1}\right)$$

Given that $\frac{1}{1+i_{ss}} = 1$ and $\frac{1}{1+\pi_{ss}} = 1$ since $\pi_{ss} = 0$ (Sims (2017)), the log linearized Euler equation becomes:

$$\tilde{C}_t - E\tilde{C}_{t+1} = -\frac{1}{\sigma}(\tilde{\imath}_t - E\tilde{\pi}_{t+1}) \quad \text{(C3.4)}$$

- **Taylor rule and monetary shock**

$$\frac{R_{ss}}{R_{ss}}(1+\widetilde{R_t}) = \left(\frac{R_{ss}}{R_{ss}}\right)^{\gamma_R}\left(\frac{\pi_{ss}}{\pi_{ss}}\right)^{\gamma_\pi(1-\gamma_R)}\left(\frac{Y_{ss}}{Y_{ss}}\right)^{\gamma_\pi(1-\gamma_R)}\Big[1+\gamma_R\tilde{R}_{t-1} + \gamma_\pi(1-\gamma_R)\tilde{\pi}_t + \gamma_Y(1-\gamma_R)\tilde{Y}_t + \tilde{S}_t^m\Big]$$

$$\widetilde{R_t} = \gamma_R\tilde{R}_{t-1} + (1+\gamma_R)\big(\gamma_\pi\tilde{\pi}_t + \gamma_Y\tilde{Y}_t\big) + \tilde{S}_t^m \quad \text{(C3.5)}$$

$$\tilde{S}_t^m = \rho_m\tilde{S}_{t-1}^m + \epsilon_{m,t} \quad \text{(C3.6)}$$